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Subcritical bubbles near the phase space domain wall

JUKKA SIRKKA[1]
and
IIRO VILJA[2]
Department of Physics, University of Turku
FIN-20500 Turku, Finland

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Abstract

We study the subcritical bubble formation near the phase space domain wall. We take into account that the phase of the scalar field can vary using complex U(1) symmetric field and a phenomenological potential with cubic term responsible to symmetry breaking. We show that the presence of the domain wall induces subcritical bubbles so that their formation rate near the wall is considerably larger than far of it. The allowed deviations of the phases of new bubbles are so large that they prevent the system from induced nucleation.

[1]sirkka@utu.fi; [2]vilja@utu.fi

The study of cosmological phase transitions have received considerable attention in literature since the possibility of electroweak baryogenesis was introduced [1]. In that context the main effort has been performed considering the critical configuration (bubble) formation which are stable, expanding broken phase domains by their own. Less attention has been paid to the vast subcritical bubble formation during the period of the metastable state, because they have been supposed to have no effect to the phase transition itself. Also it has become clear [2, 3, 4] that for physical Higgs mass, heavier that 60 GeV, the electroweak phase transition is only weak. In the context of electroweak phase transition, subcritical bubbles were discussed first time by [5] and it has been also shown [6] that phase equilibrium can be reached at weak enough phase transitions. Kinetics of subcritical bubbles has been studied in [7] where phase mixing above the critical temperature was investigated. More recently numerical simulations of phase mixing in a 2+1 –dimensional model [8] and in 3+1 –dimensional model [9] have been performed.

When the electroweak phase transition is weak enough and the formation rate of broken phase configurations is large enough, there are large number of subcritical bubbles present at and below the critical temperature. It can happen that a collection of subcritical bubbles form a region with a size of a critical bubble before they shrink away [10]. Therefore the phase transition may be triggered by clustering the broken phase regions. This mechanism of producing phase transition is called induced nucleation.

In Ref. [10] it was demonstrated that in a simple model induced nucleation is indeed possible for a large range of parameter values of the scalar potential. The growth of a (spherical) region of broken phase was approximated to happen layer by layer by nucleating subcritical bubbles. As noted there, however, the probability that the region of broken phase grows to the size of a critical bubble strongly depends on the relative phases of the subcritical bubbles. This effect was accounted in [10] by introducing a parameter which describes the probability that the phase of the nucleated subcritical bubble is correlated with the phase of the pre-existing bubble. Its origin, however, was not further analyzed there.

In the present paper we analyze the subcritical nucleation near phase boundary. In particular, we concentrate to the question, how phases of newly formed bubble and pre-existing broken phase domain correlate. We assume that the relative velocities between the bubbles are negligible. In opposite case, the relative phase of colliding bubbles would be washed out by thermal processes with a rate which is presumably much larger than in the case of bubbles in rest, leading effectively to the case where the phases of the bubbles are correlated.

One can also consider the effect of the distance u of the subcritical bubble from the pre-existing one. If u is small, the newly formed bubble overlaps significantly with

the pre-existing one, causing the spatial phases to be correlated. As u increases, the spatial phase difference becomes more probable.

We work with a $3+1$ –dimensional phenomenological model with a complex scalar Φ having the 3 –dimensional action

$$S_3[\Phi] = \int d^3x \, [|\nabla\Phi|^2 + V(\Phi)], \tag{1}$$

where the phenomenological potential is given by

$$V[\Phi] = m(T)^2|\Phi|^2 - \frac{2\sqrt{2}}{3}\alpha T|\Phi|^3 + \lambda|\Phi|^4. \tag{2}$$

Generally, the non-zero maximum v_- and minimum v_+ of the potential (below the critical temperature T_c) are given by

$$v_\pm = \frac{\alpha T}{2\lambda}\left(1 \pm \sqrt{1 - \frac{8}{9}\bar{\lambda}}\right), \tag{3}$$

where

$$\bar{\lambda} = \frac{9\lambda m(T)^2}{2\alpha^2 T^2}. \tag{4}$$

The parameter $\bar{\lambda}$ is less than unity at temperatures below T_c.

Generally there exist subcritical bubbles with various sizes and forms. We assume that a typical subcritical bubble has a Gaussian, spherically symmetric modulus

$$\Phi_b(x, y, z) = v_+(T)e^{-2\mathbf{x}^2/l^2}e^{i\chi(\mathbf{x})}, \tag{5}$$

where the real function χ determines the phase of the bubble. We use this configuration to represent all subcritical bubbles. Subcriticality implies that the subcritical bubble diameter $l \ll R_c$, the critical bubble radius. Supposing that the pre-existing broken phase domain (bubble) is large enough compared to the subcritical bubble its wall can be approximated to be planar. Thus the background configuration is given by

$$\Phi_{bg}(z) = v_+(T)e^{-2\theta(z)\,z^2/l^2}, \tag{6}$$

where θ is the step function. The formation rate of subcritical bubbles in the presence of Φ_{bg} at the distance u is determined by the conditional probability and expressed as

$$\Gamma_V[\chi] \simeq T^4\, e^{-\beta(S_3[\Phi_{bg}+\Phi_b^u]-S_3[\Phi_{bg}])}, \tag{7}$$

where the superscript u denotes that the center of the bubble has been moved to the point $\mathbf{x} = (0, 0, u)$.

Now, the normalized probability density of phase configuration χ is formally given by

$$P[\chi] = \mathcal{N}^{-1}\Gamma'_V, \tag{8}$$

where

$$\Gamma'_V = e^{-\beta(S_3[\Phi_{bg}+\Phi^u_b]-S_3[\Phi_{bg}])} \tag{9}$$

and $\mathcal{N} = \int \mathcal{D}\chi\,\Gamma'_V$. The evaluation of the functional integral is, of course, a hopeless task. In what follows we shall take some representative functions χ to study statistical averages of some parameters using Γ'_V as a probability density.

To be specific, we write $\chi = \delta\chi'$ where δ is a constant angle representing the size of the fluctuations and χ' is one of the functions $\chi_1(z) = 1/2(1+\tanh(z-u))$, $\chi_2(z) = \theta(z)\theta(u-z)z/u + \theta(z-u)$ or $\chi_3(z) \equiv 1$. Note that χ_1 and χ_2 have asymptotic values 0 and 1 corresponding to the limits $z \to -\infty$ and $z \to \infty$, respectively. The function χ_3 can be thought to be the limiting case in such a class of functions. Thus we can consider Γ'_V as the probability density of the random variables u and δ with the normalization factor

$$\mathcal{N}' = \int du\, d\delta\, \Gamma'_V[\delta\chi'] \tag{10}$$

Note, that the integrand is an even function of δ. The parameter δ can have any values in the cases of χ_1 and χ_2 but by periodicity it is limited between $-\pi$ and π in the case of χ_3.

The next step is to determine the maximal distance u_{max}. In general u is just a free parameter, but e.g. in the case of induced nucleation the new bubbles can not be formed arbitrary distant from the wall. One has to require that the new bubbles join to the pre-existing one forming one connected domain. Let us for the moment assume that the all bubbles, both the pre-existing one and the newly formed ones, have same phase; an assumption which certainly gives a maximal distance. Suppose also that the pre-existing bubble has been formed layer by layer from subcritical bubbles. By packing subcritical bubbles so that the minimum in the middle of the bubbles the field value is larger that v_-, which is required that the field at that point would roll down to the broken minimum, not towards unbroken one, one finds maximal possible u

$$\frac{u_{max}}{l_c} = \frac{2}{3}\left[2\ln\left(\frac{4v_+}{v_-}\right)\right]^{1/2}, \tag{11}$$

corresponding to the tightest (lattice) packing of spheres in 3 –dimensions, face centered cubic lattice [11]. However, at T_c, where $v_+/v_- = 2$ simple cubic lattice is equally good leading to same $u_{max} = 1.36 l_c$ but for $v_+/v_- > 2$ it results a smaller one. The mean phase fluctuation size $\bar{\delta}$ can now be given by

$$\bar{\delta}^2 = \int d\delta \int_0^{u_{max}} du\, \delta^2 P(u,\delta), \tag{12}$$

and similarly for $\bar{u}$.

We are mostly interested in weak phase transitions where the actual transition temperature T_f is close to the critical temperature T_c. Thus we, as an approximation, perform the analysis at T_c except in some formulas where the tiny difference between T_c and T_f is crucial. At small supercooling limit T_f is determined by [6]

$$S_3^b/T = \frac{\alpha}{\lambda^{3/2}} \frac{2^{9/2}\pi\bar{\lambda}^{3/2}}{3^5(\bar{\lambda}-1)^2} \simeq 150 \tag{13}$$

leading to $\bar{\lambda} = 1 - 0.0442\alpha^{1/2}/\lambda^{3/4}$. Small supercooling limit is valid if $1 - \bar{\lambda} \ll 1$, i.e. $\alpha/\lambda^{3/2} \ll 500$. The minimum at T_f is given by

$$-\epsilon \equiv V(v_+(T_f), T_f) \simeq -0.00218 \frac{\alpha^{9/2}}{\lambda^{15/4}} T_c^4. \tag{14}$$

At the critical temperature the non-zero minimum of the potential reads

$$v_+(T_c) = \frac{\sqrt{2}}{3}\frac{\alpha}{\lambda} T_c \tag{15}$$

and the mass is given by

$$m(T_c)^2 = \frac{2}{9}\frac{\alpha^2}{\lambda} T_c^2. \tag{16}$$

The correlation length in symmetric phase is

$$l_c = \frac{1}{m(T)} \tag{17}$$

and, although all sizes of subcritical bubbles exists in the symmetric phase, we use a representative subcritical bubble[6, 10] with $l = l_c$.

The effect of approximation $T_f \simeq T_c$ and use of $l = l_c$ is that the quantity βS_3 depends on the potential parameters only through the combination $\gamma \equiv \alpha/\lambda^{3/2}$. The numerical evaluation of the action shows, provided that γ is not too close to zero i.e. the phase transition is not too weak, that the main contribution to the averages $\overline{\delta}^2$ and $\bar{u}$ comes from the region where δ is close to zero and u is close to u_{max} in the cases of functions χ_1 and χ_2. But in the case of χ_3 the averages receive also a significant contribution from the region where δ is close to π and u is close to zero, as shown in Fig. 1. This is to be interpreted that in the case of the constant phase there are in average two kinds of subcritical bubbles: those having almost opposite phase as the pre-existing bubble and lying near the wall and those having almost the same phase as the pre-existing bubble and lying as far as possible from the wall. For χ_1 and χ_2 the averages are plotted in Fig. 2.

By comparing the average formation rate $\overline{\Gamma}_V'$ of subcritical bubbles near the domain wall to the rate Γ_s of a single bubble not influenced by the wall given by [10] $\Gamma_s \propto e^{-2.06\gamma}$

we find that for values $\gamma > .5$ the rate is significantly larger near the domain wall, Fig. 3. Thus the domain wall is shielded by the subcritical bubbles nearby it. The shielding is even stronger for larger values of γ. This would enhance the induced nucleation by making subcritical bubble nucleation easier. However, the relative phases of the bubbles do not correlate any more and therefore we have to estimate the influence of the phase differences.

The influence of phase deviations of the size $\bar{\delta}$ to the bubble dynamics can be most easily analyzed within the context of thin wall approximation valid in weak phase transition case. Thin wall approximation requires that the maximum of the potential $V_{max} \gg \epsilon$, which implies $\alpha/\lambda^{3/2} \ll 10$ in accordance with small supercooling limit. The critical bubble radius R is obtained by extremizing the bounce action

$$B = -\frac{4}{3}\pi\epsilon_{eff}R^3 + 4\pi\sigma R^3, \tag{18}$$

where the surface tension in thin wall approximation is given by

$$\sigma = \frac{2\sqrt{2}\alpha^3}{91\lambda^{5/2}}T_c^3. \tag{19}$$

Usually ϵ_{eff} is simply given by Eq. (14) but now it is affected by phase fluctuations in the broken phase, so that

$$\epsilon_{eff} = \epsilon - \Delta\epsilon, \tag{20}$$

where

$$\frac{4}{3}\pi R^3\Delta\epsilon = \langle \int_{\mathbf{x}\in B(0,R)} d^3x\, |\nabla\Phi_R(\mathbf{x})|^2\rangle. \tag{21}$$

Φ_R is now a random configuration having modulus v_+ and a varying phase between values $-\bar{\delta}$ and $\bar{\delta}$.

If we suppose that the typical size of the spatial extension of the phase fluctuations is $\bar{u}$, as can be done in the cases of χ_1 and χ_2, we are able to calculate $\Delta\epsilon$. When the maximum values of phase fluctuations are situated at the points $\mathbf{x} = (n_x\bar{u}, n_y\bar{u}, n_z\bar{u}),\ n_x,\ n_y,\ n_z \in \mathbf{Z}$ the calculation can be reduced to a unit lattice where $n_x,\ n_y,\ n_z = 0,\ 1$. Taking $\Phi_R = e^{i\bar{\delta}\chi(\mathbf{x})}$ the expectation value $\Delta\epsilon$ can be now expressed in terms of the unit lattice:

$$\frac{4}{3}\pi R^3\Delta\epsilon = \frac{4}{3}\pi v_+^2\bar{\delta}^2\left(\frac{R}{\bar{u}}\right)^3\langle \int_{\mathbf{x}\in[0,\ \bar{u}]^3} d^3x\,(\nabla\chi)^2\rangle. \tag{22}$$

At each corner we associate equally distributed independent random variables f_{n_x,n_y,n_z} with values ±1 and correlations $\langle f^N_{n,n,n}\rangle = \frac{1+(-1)^N}{2}$. Moreover, we choose the phase

configuration in the unit bravais lattice to be the simplest possible polynomial, i.e. the Lagrangian interpolation polynomial

$$\chi(\mathbf{x}) = -\sum_{n_x,n_y,n_z=0,\ 1} f_{n_x,n_y,n_z} \prod_{\substack{k_x,\,k_y,\,k_z\,=\,0,\ 1\\ k_x \neq n_x\\ k_y \neq n_y\\ k_z \neq n_z}} (-1)^{k_x+k_y+k_z}(k_x - \frac{x}{\bar{u}})(k_y - \frac{y}{\bar{u}})(k_z - \frac{z}{\bar{u}}).$$

An easy calculation yields now

$$\Delta\epsilon = \frac{8v_+^2}{3\bar{u}^2}\,\bar{\delta}^2\,. \tag{23}$$

Because from Eq. (18) one obtains the critical bubble radius

$$R_c = \frac{3\sigma}{\epsilon_{eff}} \tag{24}$$

the critical bubble radius tends to infinity if $\epsilon_{eff} \to 0$. Hence a general requirement for existence of the critical bubble can be stated to be $\epsilon_{eff} > 0$, i.e.

$$\bar{\delta} < 0.129\frac{\bar{u}}{l_c}\left(\frac{\alpha}{\lambda^{3/2}}\right)^{1/4}. \tag{25}$$

Thus Eq. (25) expresses an overall upper bound for mean phase fluctuations inside a critical bubble during the cosmological electroweak phase transition in terms of phenomenological parameters and fluctuation correlation length. Comparing Eq. (25) and the Fig. 2 one can conclude that it is not possible for any size of critical bubble to survive from the effects of phase fluctuations. One should have a mechanism which would damp more efficiently the variations of δ.

In the case of χ_3 the energy change has to be calculated differently. Because the small, opposite phase $\delta = \pi$ bubbles can be approximated to be isolated in the $\delta = 0$ phase, the energy change is approximated by $\delta\epsilon = \frac{E_2}{v_1+v_2}$, where E_2 is the energy of single wrong-phase bubble and v_1 and v_2 are the volumes of $\delta = 0$ and $\delta = \pi$ bubbles, correspondingly. A calculation yields an lower bound for $\delta\epsilon > 0.0595\gamma$. Requiring $\epsilon_{eff} > 0$ we obtain $\alpha\lambda^{1/2} > 0.55$ or, because $\gamma \ll 10$, $\lambda \gg 0.23$ which can hardly be true. Hence also χ_3 case confirms the conclusion that the phase deviations are too large for induced nucleation to be possible.

In the present paper we have analyzed the subcritical bubble nucleation near a domain wall. The wall itself induces nucleation nearby it so that the subcritical bubble formation rate is larger close the wall than far of it. It also appears that deviations of the phase of the field can be remarkably large and prohibits the system from induced nucleation. Note, that if the size of typical subcritical bubble is smaller that the correlation length as suggested lately [12], the bound (25) would be even stricter. Also those subcritical bubbles having very small amplitude compared to v_+ may have effect

to the large amplitude configurations. In realistic theories, like the Standard model where the symmetry group is larger, the actual numbers for $\bar{\delta}$ and $\bar{u}$ would surely alter but, because they have more degrees of freedom, the qualitative results are likely to hold.

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Figure captions

Figure 1. The fraction of 500 randomly generated points in the δ-u plane which pass the limit $\beta S_3 < 1$ with $\gamma = 1$. Open circles correspond to the case of χ_1 and filled circles to the case of χ_3.

Figure 2. The averages $\bar{\delta}[\chi_1]$ (solid line), $\bar{\delta}[\chi_2]$ (dashed line), $\bar{u}[\chi_1]$ (dotted line) and $\bar{u}[\chi_2]$ (dash-dotted line) as a function of γ. For $\gamma = 0$ the average $\bar{\delta}$ diverges as a function of the upper limit of the δ-integration. The upper limit used in the plot is $\delta_{max} = 10^2$. Studying the dependence of the upper limit of the integration numerically reveals that it is wery weak for $\gamma \gtrsim 0.1$, implying that only those values of $\bar{\delta}$ with γ larger than that limit are reliable. Note that $\gamma = 0$ implies that the scalar potential has only the symmetric phase minimum $|\Phi| = 0$.

Figure 3. The average formation rate $\overline{\Gamma}'_V$ of subcritical bubbles near the domain wall (solid line) and the rate $\Gamma_s = \exp(-2.06\gamma)$ of a single bubble not influenced by the wall (dashed line).

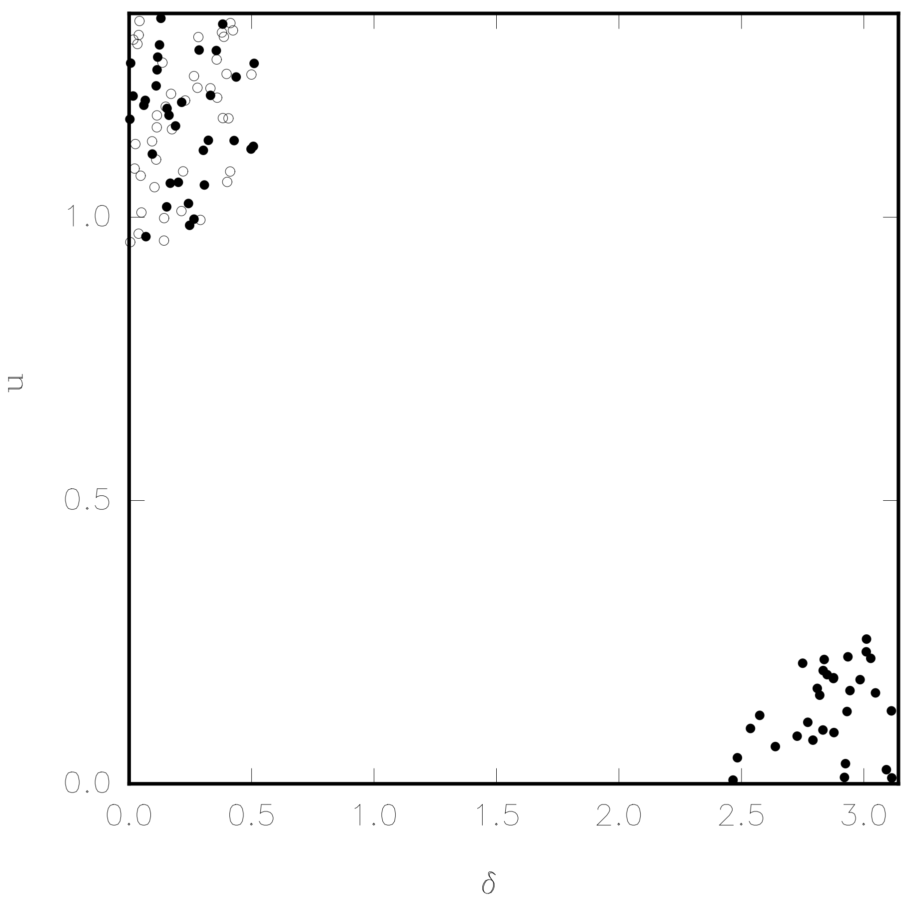

Fig. 1

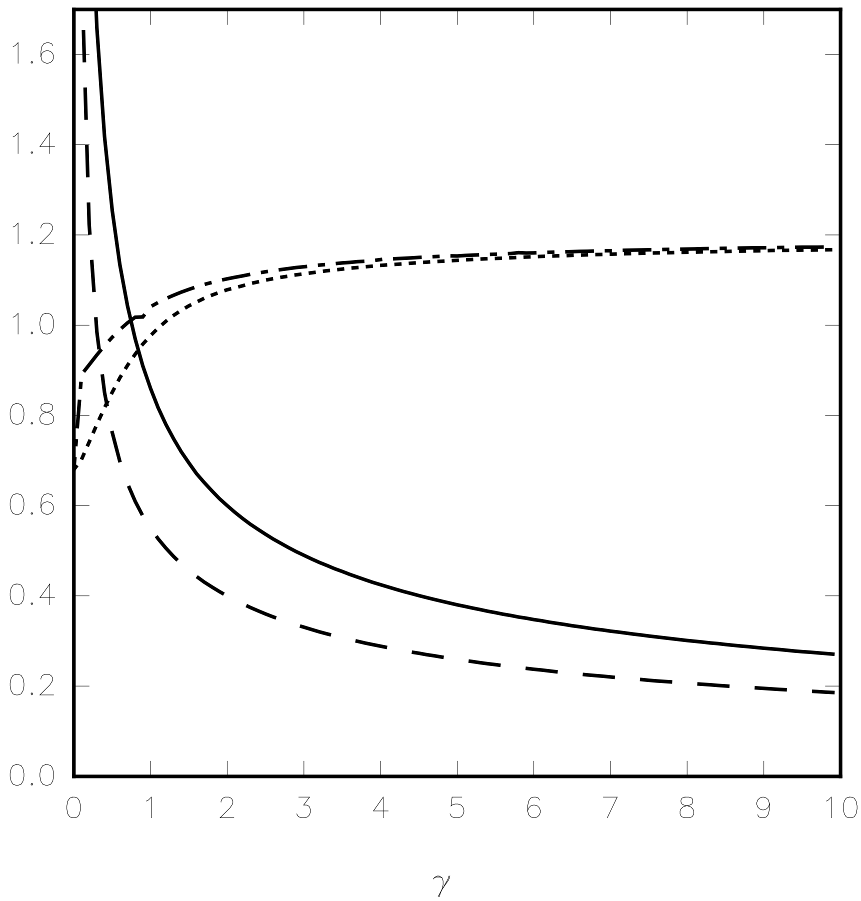

Fig. 2

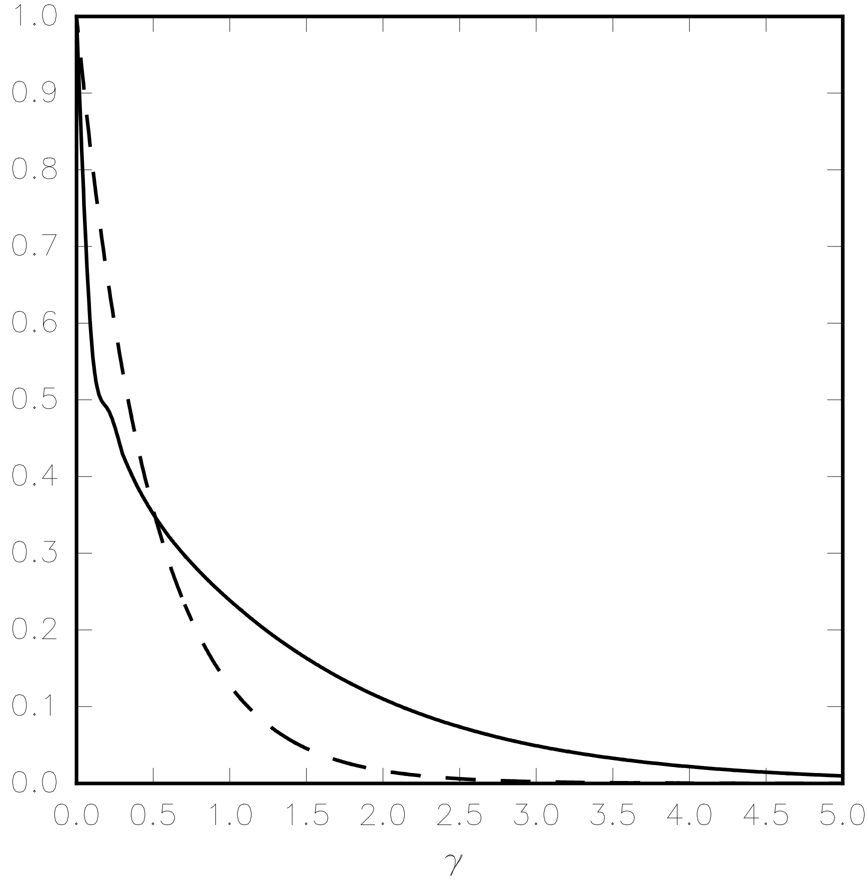

Fig. 3